Black-hole interiors and strong cosmic censorship

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Strong cosmic censorship holds that given suitable initial data on a spacelike hypersurface, the laws of general relativity should determine, completely and uniquely, the future evolution of the spacetime. Here it is argued that while strong cosmic censorship is enforced for all black holes residing in asymptotically flat spacetime, it is violated (within the classical formulation of general relativity) for some black holes residing in non asymptotically flat spacetime. It is suggested that the semi-classical formulation of general relativity might enforce strong cosmic censorship.

I. INTRODUCTION

The basic question underlying the theoretical study of black-hole interiors is "what is the structure of spacetime inside a realistic black hole?". A lot of progress has been made during the last few years toward answering this question, as will be obvious from the other contributions to these proceedings. The question I wish to consider in this contribution is the following, which is at once much more focused and much more fundamental: "Do the laws of general relativity uniquely determine the structure of spacetime inside the black hole, given suitable initial data placed at the onset of gravitational collapse?". I will argue that the evidence points to a negative answer in the case of the purely *classical* laws, but that there is hope for a positive answer in the framework of the *semi-classical* laws. This contribution is based mostly on a 1992 paper co-authored with Patrick Brady [1] and a 1995 paper co-authored with Draza Marković [2]. However, the point of view expressed here is entirely my own, and Patrick and Draza should not be held liable!

The scope of the question should be clarified before attempting to answer it. What is at stake here is the *global* structure of spacetime, that is, the full characterization of physical fields, including the metric, everywhere and at all times. The well-posedness of the initial value problem in general relativity [3] guarantees only that given suitable data on an initial surface, the solution to the Einstein equations will be unique (up to diffeomorphisms) everywhere within the *domain of dependence* of the initial surface. Denoting this surface by Σ and its domain of dependence by $D(\Sigma)$, the question facing us is whether $D(\Sigma)$ coincides with M, the entire spacetime manifold. (See Fig. 1.) In other words, is there a region in the manifold which does not lie inside the domain of dependence of the initial surface? If we define the *Cauchy horizon* $H(\Sigma)$ of the initial surface to be the boundary of its domain of dependence, then the question is whether there exists a Cauchy horizon at all, and the answer is negative if $D(\Sigma) = M$.

This set of questions is usually grouped under the name *strong cosmic censorship.* The principle of strong cosmic censorship expresses the basic idea that starting from suitable initial data, general relativity should be able to predict, unambiguously, the complete future evolution of spacetime. As we have seen, a more technical way of expressing this idea is that the domain of dependence of the initial surface should be the entire spacetime manifold, or that the Cauchy horizon of the initial surface should be the empty set. A spacetime which satisfies these properties is said to be *globally hyperbolic*. A much more precise proposal for a strong cosmic censorship conjecture can be found in Wald's book [3].

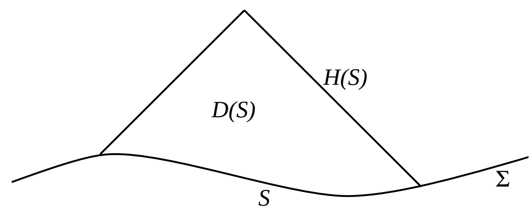

FIG. 1. A spacelike hypersurface Σ (assumed without an edge) contains a set S. The evolution of initial data put on S is determined uniquely only within $D(S)$, the domain of dependence of S. The boundary of this region is $H(S)$, the Cauchy horizon of S. Strong cosmic censorship holds that as S is made to coincide with Σ, then $D(S)$ must coincide with M, the entire spacetime manifold. Then $H(\Sigma) = \emptyset$. In the diagram, only the future parts of $D(\Sigma)$ and $H(\Sigma)$ are shown.

A more frequently discussed form of cosmic censorship is the weak form, which states, roughly, that physically realistic spacetimes should not contain any globally naked singularities. A singularity is globally naked if it can be detected by observers at arbitrarily large distances. A singularity hidden behind an event horizon is not globally naked, and such a spacetime would therefore satisfy the weak form of cosmic censorship. The strong form asks for more: If an observer traverses the event horizon, will she encounter a (locally) naked singularity? If so, physical predictions made on the basis of the initial conditions will be upset by the presence of the singularity, and strong cosmic censorship will be violated. Strong

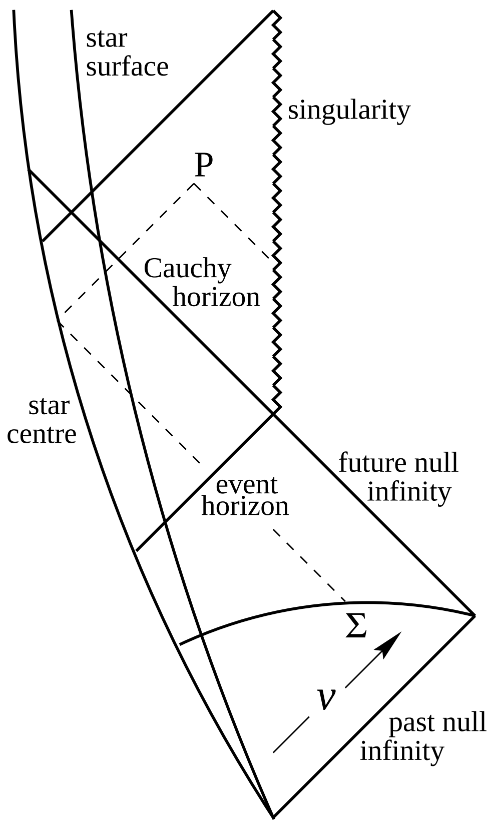

FIG. 2. Conformal diagram of the Reissner-Nordström spacetime. The ingoing branch (left-going on the diagram) of the inner horizon is a Cauchy horizon for the hypersurface Σ. This is illustrated by the fact that light rays originating from event P and propagating backward in time run into the timelike singularity at $r = 0$. In this diagram, $v = \infty$ both at future null infinity and at the Cauchy horizon.

cosmic censorship therefore holds that no singularity may be naked, even locally.

The question asked in this essay is whether the laws of general relativity enforce strong cosmic censorship.

II. BLACK HOLES IN ASYMPTOTICALLY FLAT SPACETIME

It is well known that the Reissner-Nordström, Kerr, and Kerr-Newman spacetimes contain timelike singularities inside their event horizons [4]. As these singularities are obviously naked, we must ask whether these spacetimes constitute a serious counter-example to strong cosmic censorship. I shall argue to the negative.

Figure 2 shows a conformal diagram of the Reissner-Nordström spacetime, whose metric is given by

$$ds^2 = -f dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \qquad (1)$$

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \qquad (2)$$

Here, v is a null coordinate which is constant along radial ($d\theta = d\phi = 0$), ingoing (r decreasing) null geodesics; M denotes the mass of the black hole, and Q its charge. The spacetime contains two types of horizons, located where $f = 0$. The event horizon is at $r = r_e \equiv M + (M^2 - Q^2)^{1/2}$, while the inner horizon is at $r = r_i \equiv M - (M^2 - Q^2)^{1/2}$. The diagram shows clearly that the ingoing branch of the inner horizon is a Cauchy horizon for any spacelike hypersurface Σ preceding the formation of the event horizon. The physical origin of the Cauchy horizon is also clear: predictions made at any event P to the future of the Cauchy horizon would be upset by signals originating at the timelike singularity, where physics cannot be controlled.

Clearly, the Reissner-Nordström spacetime is a counter-example to strong cosmic censorship: the spacetime contains a Cauchy horizon, beyond which the evolution of physical fields becomes ambiguous. Furthermore, the same is true for the Kerr and Kerr-Newman spacetimes, which also contain Cauchy horizons. The real question, however, is whether these spacetimes constitute a *serious* counter-example to strong cosmic censorship. By this I mean that if these spacetimes could be shown to form a set of measure zero in some topological space of black-hole spacetimes, then they could be dismissed as inconsequential. On the other hand, if black-hole spacetimes with Cauchy horizons formed an open set, then we would have to conclude that strong cosmic censorship is not enforced by general relativity.

The construction of such a topological space would be a difficult undertaking which, however, will be necessary to settle the issue. (This point was forcedly made to me by Jim Isenberg.) Here I will argue that the Kerr-Newman spacetimes should form a set of measure zero in any reasonable topological space of black-hole spacetimes.

Consider first the Reissner-Nordström spacetimes. (There is one spacetime for each value of the parameters M and Q.) These spacetimes are very special, because they result from very special initial conditions: apart from being empty of any form of matter except for a static electric field, they are exactly spherically symmetric. However, it has long been known that slight deviations from these conditions, in the form of time-dependent matter fields or gravitational waves, produce large effects at the Cauchy horizon [5–9]. More precisely, physical quantities associated with the perturbations, such as the energy density measured by a free-falling observer, diverge at the Cauchy horizon. In other words, the Cauchy horizons of the Reissner-Nordström spacetimes are *unstable* to time-dependent perturbations. The same is true for the Kerr and Kerr-Newman spacetimes. This is an indication that these spacetimes must form a set of measure zero.

This, however, is not good enough, because the perturbation analysis involves only test fields in a fixed back-

ground spacetime. What must be understood, in a non-perturbative manner, is how the spacetime itself evolves under the slightly different choice of initial conditions. This is the question that Werner Israel and I started to examine in 1989 [10]. After a lot of work, carried out most notably by Israel's group in Canada and Amos Ori's group in Israel, the answer is now clear: Both for nonrotating and rotating black holes, the spacetime will develop a null curvature singularity at the Cauchy horizon. This singularity is not of a big-crunch type as in the Schwarzschild spacetime. Instead, it is characterized by an infinite growth of the internal mass function at the Cauchy horizon, whose area remains finite. This singularity is known as the *mass-inflation* singularity. In effect, the perturbations destroy the Cauchy horizon, and replace it with a null curvature singularity. The mass-inflation scenario is now firmly established in spherical symmetry, thanks to analytic [10–13] and numerical [14,15] calculations. The evidence is somewhat less firm, but still quite good, in the case of rotating black holes [16–19].

One might ask the following question: "How is a null curvature singularity any better than a Cauchy horizon? After all, isn't predictability lost also at the singularity, because of the necessary breakdown of the classical laws there?" Eanna Flanagan provided the following answer during the workshop, which I fully endorse: The presence of a (nonsingular) Cauchy horizon inside a black hole is surprising because it signals the breakdown of the classical laws without any *local* indication that something may be wrong. For example, a free-falling observer would measure the curvature tensor to be well below Planckian values, and would never suspect that classical general relativity was about to lose predictive power. This, clearly, is not the case near a curvature singularity. In a sense, the loss of predictability occurring at a Cauchy horizon is much worse than the "mere" breakdown of the classical laws near a curvature singularity.

We may conclude that there is strong evidence that black-hole spacetimes with Cauchy horizons form a set of measure zero, because slight deviations in the initial conditions produce spacetimes whose causal structure is drastically different. In effect, slightly different initial conditions destroy the Cauchy horizon and replace it with a null curvature singularity. It is therefore tempting to suggest that the Kerr-Newman spacetimes do not constitute a serious counter-example to strong cosmic censorship. In fact, I suspect that the following statement is true: In the topological space of all asymptotically-flat black-hole spacetimes, the set of all spacetimes containing a Cauchy horizon has zero measure. Of course, no mathematically rigourous proof of this statement exists.

III. CAUCHY-HORIZON INSTABILITY

It is useful to have a clear understanding of the physical processes leading to the Cauchy-horizon instability. For simplicity, we restrict attention to the Reissner-Nordström spacetime.

We consider a simple model involving a test distribution of noninteracting massless particles. The particles originate from the region outside the black hole, move radially inward along curves of constant v, and eventually fall inside the black hole. They are described by the stress-energy tensor

$$T_{\alpha\beta} = \frac{L(v)}{4\pi r^2}\,(\partial_\alpha v)(\partial_\beta v), \tag{3}$$

where the luminosity function is given by $L(v) \sim v^{-p}$ (with p a positive constant) when $v \to \infty$, to correctly reproduce Price's inverse-power law decay of radiative fields outside the black hole [20–23]. We recall that $v = \infty$ designates both future null infinity and the Cauchy horizon (see Fig. 2).

The flux of particles is observed inside the black hole by a free-falling observer crossing the Cauchy horizon. This observer moves on a radial geodesic, has a four-velocity u^α, and measures the energy density of the particles to be $\rho = T_{\alpha\beta}u^\alpha u^\beta = L(v)\dot{v}^2/4\pi r^2$, where $\dot{v} \equiv u^v$. A simple calculation reveals that $\dot{v} \sim |\tilde{E}|\exp(\kappa_i v)$ when $v \to \infty$, where $\tilde{E} \equiv -u_v$ is the observer's energy parameter, and

$$\kappa_i = \frac{1}{2}\left|\frac{df}{dr}\right|_{r=r_i} \tag{4}$$

is the *surface gravity* of the inner horizon. Substitution yields

$$\rho \sim \frac{|\tilde{E}|^2}{4\pi {r_i}^2}\,v^{-p}e^{2\kappa_i v}, \qquad v \to \infty. \tag{5}$$

Thus, the measured energy density diverges as the observer reaches the Cauchy horizon. In terms of v, this divergence is exponential; in terms of τ, the amount of proper time left before reaching the Cauchy horizon, $\rho \sim 1/\tau^2$. The physical interpretation is that the particles pile up at the Cauchy horizon, which is a surface of infinite blueshift. Consequently, the energy density diverge there. It is perhaps not surprising that a full backreaction calculation reveals the existence of a null curvature singularity at the Cauchy horizon.

The extremal case ($|Q| = M$) requires a separate treatment, because $\kappa_i = 0$ for this spacetime. It turns out that in this case, the Cauchy horizon is *stable* to time-dependent perturbations [24]. The reason is that although an infinite blueshift still occurs at the Cauchy horizon, the blueshift factor diverges only as a power of v instead of exponentially. Because $L(v)$ decays with a larger power, the energy density stays finite. This observation should not affect our conclusions, as presumably, extremal black holes also form a set of measure zero of spacetimes.

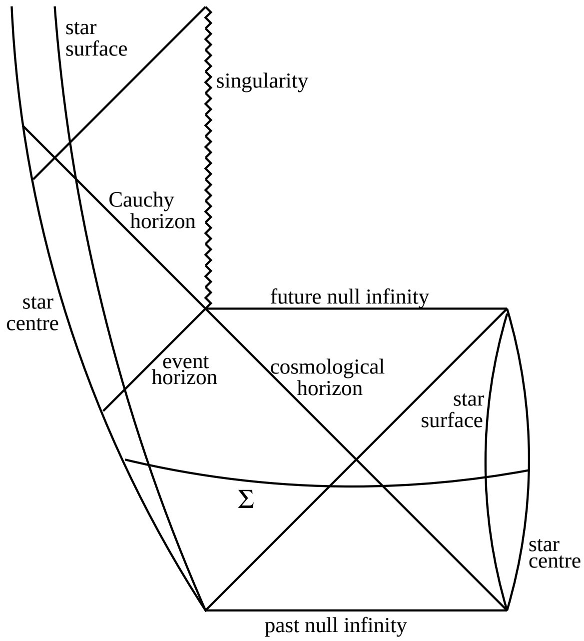

FIG. 3. Conformal diagram of the Reissner-Nordström-de Sitter spacetime. The ingoing branch of the inner horizon is a Cauchy horizon for the hypersurface Σ. In this diagram, $v = \infty$ both at the cosmological horizon and at the Cauchy horizon.

IV. BLACK HOLES IN NON ASYMPTOTICALLY FLAT SPACETIME

While it appears highly plausible that strong cosmic censorship is enforced for black holes residing in asymptotically flat spacetime, the same cannot be said for black holes residing in asymptotically de Sitter spacetime.

Consider first the Reissner-Nordström-de Sitter spacetime [25], whose metric is given by Eq. (1) with

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{1}{3}\Lambda r^2, \tag{6}$$

where Λ is the cosmological constant, assumed to be positive. This spacetime contains three types of horizons, situated at the three positive roots of f. The cosmological horizon is located at $r = r_c$, the largest root. The black-hole event horizon is at $r = r_e$, and the inner horizon at $r = r_i$ is also a Cauchy horizon for any spacelike hypersurface Σ lying outside the black hole. A conformal diagram of this spacetime is presented in Fig. 3. We see from the diagram that the cosmological horizon plays here the same role that future null infinity played in the diagram of Fig. 2.

The major difference between this class of spacetimes and the Reissner-Nordström class is that here, the parameters $\{M, Q, \Lambda\}$ can be chosen so that the Cauchy horizon is *stable* to time-dependent perturbations. As was first shown by Brady and myself [1] on the basis of the simple argument presented in Sec. V, and then confirmed by Mellor and Moss [26] on the basis of a complete perturbation analysis, this happens whenever

$$\kappa_i \le \kappa_c, \tag{7}$$

where κ_i is the surface gravity of the inner horizon, defined by Eq. (4), and $\kappa_c = \frac{1}{2}|df/dr|(r_c)$ the surface gravity of the cosmological horizon. Equation (7) defines a small but finite region of the parameter space $\{M, Q, \Lambda\}$. This region is described in detail in Chambers' contribution to these proceedings. That the region must be small can be seen as follows (I thank Ian Moss for providing me with this argument): In situations close to realistic, the cosmological horizon would be at a very large radius, making κ_c very small; to have κ_i smaller than this requires the black hole to very nearly extremal, $|Q| = M(1-\epsilon)$, with ϵ a very small positive number.

Equation (7) implies Cauchy-horizon stability not only for the Reissner-Nordström-de Sitter spacetimes, but also for the entire Kerr-Newman-de Sitter class. This was established by Chambers and Moss [27], who also showed that Eq. (7) defines a small but finite region of the parameter space $\{M, Q, a, \Lambda\}$, where a is the black hole's rotation parameter. Again, this is discussed in more detail in Chambers' contribution. Equation (7) was also shown to imply stability in a fully nonperturbative analysis restricted to spherical symmetry [28].

If the open region of parameter space happens to correspond to an open set in the topological space of black-hole spacetimes, then we would be forced to conclude that strong cosmic censorship is *not* enforced by general relativity. There is, of course, no rigourous proof that the spacetimes for which Eq. (7) is satisfied do indeed form an open set, but I would regard this as highly plausible.

This example of an apparent violation of strong cosmic censorship relies on the presence of a cosmological constant in the field equations, something which may be distasteful to some. However, another example was recently discovered by Horowitz and Sheinblatt [29], and it does not require a cosmological constant. (I thank Gary Horowitz for pointing out this work to me.) These authors consider two oppositely charged black holes, each uniformly accelerating in a background magnetic field. The solution to the Einstein-Maxwell equations describing this spacetime is known as the Ernst solution [30], and the causal structure of this spacetime is essentially identical to that of the Reissner-Nordström-de Sitter spacetime, except for the fact that the cosmological horizon is replaced by an acceleration horizon. Another similarity is that the spacetime is also not asymptotically flat. Horowitz and Sheinblatt show that the Cauchy horizon of the Ernst spacetime is *stable* whenever $\kappa_i \le \kappa_a$, where κ_a is the surface gravity of the acceleration horizon. Again, this inequality defines a finite region of parameter space, and it is tempting to suggest that this corresponds to an open set in the topological space of black-hole spacetimes. If this were true, then strong cosmic censorship would be violated also for this class of spacetimes.

As we see, the fact that strong cosmic censorship might be violated for certain black-hole spacetimes is not necessarily due to the presence of a cosmological constant in the field equations. Instead, the essential features seem to be the presence of a "large" horizon and the absence of asymptotic flatness. I would conjecture the following statement: In the topological space of all black-hole spacetimes, including those which are not asymptotically flat, the set of all spacetimes containing a Cauchy horizon is open. If this statement is true, then strong cosmic censorship is not enforced by general relativity.

V. CAUCHY-HORIZON STABILITY

It is easy to understand why the Cauchy horizon of the Reissner-Nordström-de Sitter spacetime is stable when $\kappa_i \leq \kappa_c$. We consider once again the simple model of Sec. III, involving noninteracting massless particles described by the stress-energy of Eq. (3).

In Sec. III, the luminosity function $L(v)$ was taken to decay as an inverse-power law in order to correctly reproduce the behaviour of radiative fields in the Reissner-Nordström spacetime. This choice, however, is not appropriate for the Reissner-Nordström-de Sitter spacetime [31]. To determine the correct behaviour of $L(v)$ we shall calculate ρ_c, the energy density of the infalling particles as measured by a free-falling energy crossing the cosmological horizon. We recall that $v = \infty$ designates both the cosmological horizon and the Cauchy horizon (see Fig. 3). The steps are the same as in Sec. III, and we find

$$\rho_c \sim \frac{\tilde{E}_c^2}{4\pi {r_c}^2}\, L(v) e^{2\kappa_c v}, \qquad v \to \infty, \tag{8}$$

where $\tilde{E}_c$ is the observer's energy parameter. We demand that the observer measure a finite, nonvanishing energy density as she crosses the cosmological horizon. This requires $L(v)$ to decay exponentially:

$$L(v) \sim K e^{-2\kappa_c v}, \qquad v \to \infty, \tag{9}$$

where K is a constant. While Eq. (9) serves mostly to remedy the bad behaviour of the coordinates v and r at the cosmological horizon, it also expresses the fact that this horizon is a surface of infinite redshift.

Substituting Eq. (9) into the calculation of Sec. III reveals that the energy density of the particles, as measured by an observer crossing the *Cauchy* horizon, is given by

$$\rho_i \sim \frac{|\tilde{E}_i|^2}{4\pi {r_i}^2}\, K e^{2(\kappa_i - \kappa_c)v}, \qquad v \to \infty. \tag{10}$$

We see that this quantity diverges when $\kappa_i > \kappa_c$, but that it stays finite (in fact, goes to zero) when $\kappa_i \leq \kappa_c$. This is just the condition expressed by Eq. (7). The interpretation is clear. The factor $\exp(2\kappa_i v)$ comes from the infinite blueshift occurring at the inner horizon, while the factor $\exp(-2\kappa_c v)$ comes from the infinite redshift occurring at the cosmological horizon. When $\kappa_i > \kappa_c$ the blueshift wins over the redshift, and the Cauchy horizon is unstable. On the other hand, when $\kappa_i \leq \kappa_c$ the redshift wins, and the Cauchy horizon is stable.

VI. QUANTUM EFFECTS

We now consider the quantum stability of the Reissner-Nordström-de Sitter spacetime. This question is addressed by admitting the existence of quantized matter fields in the spacetime, and examining the behaviour of $\langle T^{\alpha\beta}\rangle$, the renormalized expectation value of their stress-energy tensor, near the Cauchy horizon. This is a difficult question, even when no attempt is made to take into account the spacetime's response to the quantum stress-energy tensor. The difficulty resides in the calculation of $\langle T^{\alpha\beta}\rangle$, even in a fixed spacetime possessing spherical symmetry. The difficulty is even more acute in our case, because (as will become clear below) the quantum state cannot be chosen among the standard ones, such as the Hartle-Hawking or Unruh vacua.

We shall therefore consider a simpler problem, that of quantizing matter fields in a two-dimensional version of the Reissner-Nordström-de Sitter spacetime, with metric

$$ds^2 = -f\, du dv, \tag{11}$$

where f is given by Eq. (6) and the null coordinate u is defined by $du = dv - 2f^{-1}dr$. (It should be noted that the coordinates u and v cannot be extended across the event horizon. This will not be a problem for this calculation.) In two dimensions, $\langle T^{ab}\rangle$ can be computed explicitly [32]; we will do so for a conformally invariant scalar field.

The calculation of $\langle T^{ab}\rangle$ starts with the trace-anomaly equation [33],

$$\langle T\rangle = \alpha R, \tag{12}$$

where $\alpha = (24\pi)^{-1}$ and $R = -f''$ is the Ricci scalar associated with the metric of Eq. (11); primes indicate differentiation with respect to r. This equation immediately gives us one component of the stress-energy tensor,

$$\langle T_{uv}\rangle = \frac{\alpha}{4}\, f f''. \tag{13}$$

The others are obtained by integrating the conservation equations, $\langle T^{ab}\rangle_{;b} = 0$. A straightforward calculation yields

$$\langle T_{uu}\rangle = -\frac{\alpha}{2}\Big[F(r) - A(u)\Big] \tag{14}$$

and

$$\langle T_{vv}\rangle = -\frac{\alpha}{2}\Big[F(r) - B(v)\Big], \tag{15}$$

where

$$F(r) = \frac{1}{4}\left(f'^2 - 2ff''\right), \tag{16}$$

while $A(u)$ and $B(v)$ are arbitrary functions which serve to define the state of the quantum field. We demand that this state be regular on the cosmological and event horizons, so that the quantum stress-energy tensor will also be regular there.

To see what requirements must be made on $A(u)$ and $B(v)$, we construct $\langle\rho\rangle \equiv \langle T_{ab}\rangle u^a u^b$, the energy density as measured by a free-falling observer with two-velocity u^a. This has components

$$u^v = \frac{\tilde{E} \pm (\tilde{E}^2 - f)^{1/2}}{f} \tag{17}$$

and

$$u^u = \frac{\tilde{E} \mp (\tilde{E}^2 - f)^{1/2}}{f}, \tag{18}$$

where $\tilde{E}$ is the observer's energy parameter, and the upper (lower) sign is chosen if r is increasing (decreasing) along the world line. We obtain

$$\langle\rho\rangle = -\frac{\alpha}{2f^2}\Big[(2\tilde{E}^2 - f)(2F - A - B) \pm 2\tilde{E}(\tilde{E}^2 - f)^{1/2}(A - B) - f^2 f''\Big]. \tag{19}$$

We now want to evaluate $\langle\rho\rangle$ near the horizon $r = r_j$, where r_j stands for either r_c (cosmological horizon), r_e (event horizon), or r_i (Cauchy horizon). A straightforward calculation shows that near $f = 0$, Eq. (19) reduces to

$$\langle\rho\rangle = -\frac{\alpha}{2f^2}\Big\{\big[2{\kappa_j}^2 - (1 \mp \epsilon)A - (1 \pm \epsilon)B\big] \times (2\tilde{E}^2 - f) + O(f^2)\Big\}, \tag{20}$$

where $\kappa_j = \frac{1}{2}|f'(r_j)|$ is the surface gravity of the horizon under consideration, and $\epsilon \equiv \mathrm{sign}(\tilde{E})$. Notice that we have never used the detailed form of the function $f(r)$ in this calculation.

We first evaluate $\langle\rho\rangle$ at the cosmological horizon, where $r = r_c$ and $v = \infty$. An observer crossing this horizon has a positive energy parameter ($\epsilon = +1$) and moves in the direction of increasing r (upper sign). The term within the square brackets therefore reduces to $2{\kappa_c}^2 - 2B(\infty)$. If we demand

$$B(v \to \infty) = {\kappa_c}^2 + O(e^{-2\kappa_c v}), \tag{21}$$

then this term will be $O(f^2)$ and $\langle\rho\rangle$ will be well-behaved at the cosmological horizon. Thus, our choice of quantum state is restricted by Eq. (21).

Next, we evaluate $\langle\rho\rangle$ at the event horizon, where $r = r_e$ and $u = \infty$. An observer crossing this horizon has a positive energy parameter ($\epsilon = +1$) and moves in the direction of decreasing r (lower sign). The square-brackets term becomes $2{\kappa_e}^2 - 2A(\infty)$. If we demand

$$A(u \to \infty) = {\kappa_e}^2 + O(e^{-2\kappa_e u}), \tag{22}$$

then this term will be $O(f^2)$ and $\langle\rho\rangle$ will be well-behaved at the event horizon. Thus, our choice of quantum state is further restricted by Eq. (22). It is interesting to note that only the asymptotic behaviours of $A(u)$ and $B(v)$ are restricted by the regularity conditions. Otherwise, these functions are completely arbitrary, allowing for much freedom in the choice of the quantum state. A *particular* quantum state meeting the regularity requirements is the Marković-Unruh vacuum [34].

Finally, we evaluate $\langle\rho\rangle$ at the Cauchy horizon, where $r = r_i$ and $v = \infty$. Since an observer crossing this horizon moves with a negative energy parameter in the direction of decreasing r, we must use $\epsilon = -1$ and the lower sign. The square-brackets term becomes $2{\kappa_i}^2 - 2B(\infty)$ which, by virtue of Eq. (21), is $2({\kappa_i}^2 - {\kappa_c}^2)$. This gives

$$\langle\rho\rangle \sim 2\alpha\tilde{E}^2({\kappa_i}^2 - {\kappa_c}^2)\,\frac{1}{f^2}. \tag{23}$$

Thus, $\langle\rho\rangle$ diverges at the Cauchy horizon.

We have proved the following theorem:

> In two-dimensional Reissner-Nordström-de Sitter spacetime, for *any* quantum state regular on both the cosmological horizon and the event horizon, the renormalized expectation value of the stress-energy tensor of a conformally invariant scalar field diverges at the Cauchy horizon, except when $\kappa_i = \kappa_c$.

The theorem implies that the two-dimensional spacetime is quantum mechanically unstable, except for the set of measure zero of spacetimes for which $\kappa_i = \kappa_c$. A similar theorem is believed to hold also for the Ernst spacetime [29].

What does this theorem tell us about the four-dimensional world? Although we shall not go into these details here [2], physical intuition suggests that the four-dimensional spacetime must also be quantum mechanically unstable. Indeed, it appears that the quantum physics of the Cauchy-horizon instability, which is revealed by the two-dimensional calculation, is robust and does not depend on the dimensionality of spacetime nor the nature of the quantum field (its spin, conformal invariance, etc.). The quantum mechanical instability can be intuitively explained in terms of fundamental processes such as the creation of thermal quanta near horizons, and the gravitational redshifts and blueshifts that these quanta undergo. Since these processes take place equally well in four as in two dimensions, I am quite confident that the quantum stress-energy tensor will diverge also at the Cauchy horizon of the four-dimensional spacetime.

VII. CONCLUSION

I will conclude with these two statements:

- Black-hole Cauchy horizons may be classically stable if the black hole does not reside in asymptotically flat spacetime. This suggests that strong cosmic censorship is not enforced by the classical formulation of general relativity.

- Black-hole Cauchy horizons are always quantum mechanically unstable, except possibly for a set of measure zero of spacetimes. This suggests that strong cosmic censorship might be enforced by the semi-classical formulation of general relativity.

Although these statements are clearly not supported by rigourous mathematical proofs, I regard the evidence for their validity as quite compelling. (Of course, the second part of the second statement is pure speculation.)

To the extent that these statements are true, it is most intriguing that quantum physics must be invoked in order to restore the full predictive power of general relativity. Here we may notice an interesting similarity with the physics of chronology horizons [35].

And to the extent that the first statement is true, the following question remains: "Does the semi-classical formulation of general relativity really enforce strong cosmic censorship?"

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